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Numerical analysis of entropy generation in concentric curved annular ducts †

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Abstract

The entropy generation has been numerically investigated in concentric curved annular square ducts under constant wall temperature boundary condition. The problem has been assumed to be steady, hydrodynamically and thermally fully developed and incompressible laminar flow with constant physical properties. The solutions of discretized equations for continuity, momentum and energy have been obtained by using an elliptic Fortran Program based on the SIMPLE algorithm. Solutions have been achieved for i) Dean numbers ranging from 3.6 to 207.1, ii) Annulus dimension ratios of 5.5, 3.8, 2.9 and 2.36, and iii) Prandtl number of 0.7. In this regard, local entropy generation as well as overall entropy generation in the whole flow field has been analyzed in detail. Moreover, the effects of Dean number and annulus dimension ratio on entropy generation arising from the friction and heat transfer have been investigated. Accordingly, it is concluded that the effect of volumetric entropy generation that is a result of fluid frictional irreversibility can be neglected as compared with volumetric entropy generation due to heat transfer irreversibility. As Dean number increases, the distribution of volumetric entropy generation coming out from the heat transfer irreversibility is formed by the temperature field, which is depending on the curvature.

Keywords: Concentric curved annular duct; Laminar forced convection; Entropy generation, Irreversibility; Numerical solution; Exergy

1. Introduction

A shape memory concentric curved annular channels is used in applications in low Reynolds number fluid flows in low rate heat exchangers design for food processing and solar collector applications because of high heat transfer capacity. Also, as the heat transfer rate are considerably high on the concave wall of the channel and convex wall of the core, curved annular ducts can be used for battery and electronic cooling applications. Curved annular channel applications may be limited because of manufacturing difficulties at the present time but near future the curved annular ducts may be preferred because of their contribution to efficiently and effectively heating and/or cooling processes in energy systems. Laminar flow in the concentric curved annular channel is characterized by the secondary flow created by centrifugal effects in the cross-section. The nature of this phenomenon depends upon the Dean number, which is the ratio of the Reynolds number to the square root of the dimensionless radius of curvature. The secondary flow motion enhances the heat transfer however, it induces more serious pressure drop in flow field. The secondary flows passages originate principally from the interaction between the centrifugal force, the pressure gradient, and the viscous forces. The heat transfer and fluid flow in curved annular ducts with different cross-section have been investigated by researchers [1-7]. It is known that the Dean number and radius ratio highly affect the friction factor and the Nusselt number in curved circular annular ducts [2, 3]. Kucuk and Asan [6, 7] numerically investigated fully developed laminar flow under constant wall temperature boundary condition in concentric and eccentric curved annular square duct, respectively. They determined that viscous forces become more effective upon centrifugal forces while the annulus dimension ratio decreases. However, they found out that when the Dean number increases the centrifugal forces are more dominant than viscous forces. They point out that the heat transfer and friction factor are affected by curvature, annulus dimension ratio and core position. Also, they showed that the secondary flows resulting from centrifugal forces highly affect the distribution of the velocity and temperature fields.

The foregoing discussions are related with forced convection flow and thermal problems inside curved annular ducts. These studies have been restricted, from a thermodynamic point of view, to only First Law (of Thermodynamics) analyses. The contemporary trend in the field of heat transfer and thermal design is to conduct Second Law (of Thermodynamics) analyses including the design-related concept of entropy generation and its minimization [8]. This new trend is important and, at the same time, necessary if the heat transfer com-

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munity is to contribute to a viable engineering solution to the energy problems [8]. Bejan [8] focused on the different mechanisms behind entropy generation in applied thermal engineering. The generation of entropy destroys available work (or exergy) of a system. Therefore, it makes good engineering sense to focus on the irreversibility of heat transfer and fluid flow processes and try to understand the function of related entropy generation mechanisms. Bejan [9] has described the systematic methodology of computing entropy generation through heat and fluid flow in heat exchangers.

Based on the minimal entropy generation principle, optimal design of thermal systems has been widely proposed from the viewpoint of the Second Law for different channel shapes, flow types, boundary conditions and solution methods [10-23]. In view of that, most of the previous investigations on convective flows in curved rectangular ducts are restricted to the analysis based on the First Law of Thermodynamics, a recent work of Ko and Ting [10] and Ko [11] investigated the entropy generation due to laminar forced convection in a curved rectangular duct with constant wall heat flux and a single longitudinal rib mounted at midway of the heated wall, respectively. The First and Second Laws of thermodynamics were applied to the forced convection heat transfer and fluid flow inside a cylindrical annulus by Mahmud and Fraser [17]. Yari [18] analytically investigated the entropy generation in a micro annulus flow for fully developed laminar flow with uniform heat flux at the walls. Second law characteristics of heat transfer and fluid flow due to forced convection of steady-laminar flow of incompressible fluid inside channel with circular cross-section and channel made of two parallel plates were analyzed by Mahmud and Fraser [20].

In completing a review of the literature, it is found that no study on entropy generation in concentric curved annular ducts with rectangular cross section has been performed. For this reason, the present study has concentrated on both hydrodynamically and thermally fully developed laminar flow in a concentric curved annular duct with square cross-section to obtain entropy generation. The local entropy generation distributions as well as the overall entropy generation in the whole flow field have been analyzed. The effects of Dean number and annulus dimension ratios on entropy generated from frictional irreversibility and heat transfer irreversibility have been investigated in detail. In terms of the scientific and industrial benefits of this study, it is expected that this study, which includes all details on entropy generation in concentric curved annular square ducts, will aim to give detail knowledge about entropy generation due to fluid frictional irreversibility and heat transfer irreversibility, to present the effect of annulus dimension ratio on entropy generation in concentric curved annular square ducts, to show the effect of centrifugal force created by curvature on entropy generation resulting from frictional and heat transfer irreversibility and to compare the entropy generation due to fluid frictional irreversibility with entropy generation due to heat transfer irreversibility.

2. Problem statement

2.1 General assumptions

The following assumptions have been taken into consideration for the numerical solution:

- Steady-state, hydrodynamically and thermally fully developed, incompressible laminar flow.
- Constant physical properties.
- The core wall temperature which is hot ($T_i = T_h$) and the channel wall temperature which is cold ($T_o = T_c$) are constant.
- Radius of curvature (R) is large compared with the channel dimensions (R+x≈R) [6, 7, 25-28].

2.2 Governing equations and boundary conditions

The physical configuration and the coordinate system of the problem are shown in Fig. 1. The governing equations in the cross section in a curved duct and the required boundary conditions can be written as [6, 7, 24-27]:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{u}{R+x} = 0 \tag{1}$$

Momentum Equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{w^2}{R+x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{R+x}\frac{\partial u}{\partial x} - \frac{u}{(R+x)^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{R+x}\frac{\partial v}{\partial x}\right) \tag{3}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - \frac{uv}{R+x} = -\frac{R}{(R+x)\rho}\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{1}{R+x}\frac{\partial w}{\partial x} - \frac{w}{(R+x)^2}\right) \tag{4}$$

Energy Equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \frac{R}{(R+x)}w\frac{\partial T}{\partial z} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{1}{(R+x)}\frac{\partial T}{\partial x}\right] \tag{5}$$

where u, v and w are velocity components in x-, y- and z-directions, respectively, ρ is density, ν is kinematic viscosity and α is thermal diffusivity.

The model neglects all terms of the order 1/R and $1/R^2$, with the exception of the centrifugal force term [6, 7, 25-28].

The equations are subjected to the following boundary conditions at the channel and core walls:

$$u = v = w = 0\ ,\ \ T_o = T_c \ \text{ and }\ T_i = T_h \tag{6}$$

In the fully developed flow, the pressure gradient varies only in the cross-section of the curved duct. Therefore, the axial pressure gradient ∂P/∂z in Eq. (4) remains constant [6, 7, 25, 27-30]. The axial temperature gradient ∂T/∂z in Eq. (5) is also assumed constant [6, 26, 27, 29].

3. Numerical solution

The Eqs. (1)-(5) are approximated with finite difference equations by the control volume-based finite difference method for the dependent variables, u, v, w and T. The convection and diffusion terms are discretized by using the upwind scheme and the central difference scheme, respectively. The finite difference equations for the dependent variable of interest are solved by ADI (Alternating-Direction Implicit) method [31]. This method uses the Tri-Diagonal Matrix Algorithm, TDMA, making successive sweeps over the computational field. Because the pressure-correction equation is a Poisson equation, Alternating-Direction Implicit solution of the difference equations is replaced by the Stone's solution method [32]. A staggered grid system is employed in this study and the solutions are obtained by an iterative scheme. Iteration is repeated until the residuals in each equation are small enough and the relaxation factor is taken 0.5, 0.5, 1, 0.7 and 0.45 for u, v, w, T and P, respectively. To check the validity of the numerical results, grid-independent study has been performed and a uniform grid system of 100 x 100 has been chosen for all the cases in this study.

After numerically determining the velocity and the temperature fields, the volumetric entropy generation due to the heat transfer irreversibility (S_T''') and the fluid frictional irreversibility (S_P''') can be calculated by the following equations [9, 10, 11, 16]:

$$S_T''' = \frac{k}{T^2}\left(\left|\nabla T\right|\right)^2 \tag{7}$$

where k is thermal conductivity

$$S_P''' = \frac{\mu}{T}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial u}{\partial y} \tag{8}$$

where μ is dynamic viscosity.

The total volumetric entropy generation in the flow field can be obtained to be

$$S_{gen}''' = S_T''' + S_P''' \tag{9}$$

According to Bejan [9], the ratio of S_P''' and S_T''' is defined as the irreversibility distribution ratio, ϕ :

$$\phi = \frac{S_P'''}{S_T'''} \tag{10}$$

Bejan number (*Be*) proposed by Paoletti *et al.* [33] is a parameter that describes the contribution of heat transfer entropy generation on overall entropy generation, which is defined as

$$Be = \frac{S_T'''}{S_{gen}'''} \tag{11}$$

The range of *Be* is from 0 to 1; *Be* = 0 and *Be* = 1 are two limiting cases representing the irreversibility is dominated by fluid friction and heat transfer, respectively.

For evaluation of the entropy generation in whole flow field, the average entropy generation rates, $S_{P,ave}'''$, $S_{T,ave}'''$, $S_{gen,ave}'''$, in the whole curved duct are defined by

$$S_{P,ave}''' = \frac{1}{A}\iint S_P''' dxdy \tag{12}$$

$$S_{T,ave}''' = \frac{1}{A}\iint S_T''' dxdy \tag{13}$$

$$S_{gen,ave}''' = \frac{1}{A}\iint S_{gen}''' dxdy \tag{14}$$

Average irreversibility distribution ratio (ϕ_{ave}) and average Bejan number (Be_{ave}) are defined in Eqs. (15) and (16) as fallows,

$$\phi_{ave} = \frac{S_{P,ave}'''}{S_{T,ave}'''} \tag{15}$$

$$Be_{ave} = \frac{S_{T,ave}'''}{S_{gen,ave}'''} \tag{16}$$

The average velocity is calculated as

$$w_{ave} = \frac{1}{A}\iint w dxdy \tag{17}$$

where A is duct cross section area

The Reynolds number is given as

$$\mathrm{Re} = \frac{\rho w_{ave} D_h}{\mu} \tag{18}$$

where D_h is hydraulic diameter.

The Dean number is defined as fallows

$$De = \mathrm{Re}\sqrt{\frac{D_h}{R}} \tag{19}$$

4. Results and discussion

The entropy generation in the concentric curved annular square ducts under constant wall temperature boundary condition has been numerically studied by taking into consideration the above general assumptions. In order to validate the present

Table 1. Comparison of numerical results in the curved square channel for Dean number and friction factor ratio.

dP^+/dz^+	r^+	grid	Dean number (De)				
			Friction factor ratio (fRe_c/fRe_s)				
			Present Study	T.W. Gyves [26]	Cheng et al. [34]	Komiyama et al. [35]	Dong and Ebadian [30]
-3900	100	30x30	14.0	14.0	13.9	14.0	14.1
			1.01	1.01	1.01	1.00	1.00
-9000	100	30x30	29.7	29.7	29.5	29.8	–
			1.08	1.07	1.07	1.06	–
-19000	100	30x30	55.0	55.2	54.8	55.4	–
			1.23	1.21	1.22	1.20	–
-39500	100	30x30	99.1	100.6	100.0	101.8	99.1
			1.42	1.38	1.41	1.38	1.42
-70000	100	40x40	151.3	151.1	151.1	150.5	–
			1.65	1.63	1.63	1.63	–
-110000	100	40x40	214.6	210.9	202.6	209.5	201.4
			1.83	1.83	1.91	1.91	1.92

Table 2. Comparison of numerical results in the curved square channel for Nusselt number ratio.

Present study		Hwang and Chao [36]	
Dean number	Nu_c/Nu_s	Dean number	Nu_c/Nu_s
0.0	1.00	0.0	1.00
71.5	1.76	70.7	1.73
101.3	2.06	100.0	2.00
141.0	2.30	141.4	2.33
214.6	2.79	223.6	2.95

study, the results obtained are compared with the literature. The comparisons of the Dean number and friction factor ratio results obtained by Gyves [26], Dong and Ebadian [30], Cheng et al. [34] and Komiyama et al. [35] are given in Table 1 depending on the dimensionless axial pressure gradient (dP^+/dz^+) for dimensionless radius of curvature ($r^+=R/D_h$) of 100. As can be seen in Table 1, there is very close agreement with the data available in the literature for Dean number and friction factor. Also, it seems that the Nusselt number is close agreement with the previous study by Hwang and Chao [36] (Table 2).

The solutions have been performed by taking into account the following parameters: (i) the Dean number ranging from 2.8 to 207.1, (ii) the annulus dimension ratio a/b=5.5, 3.8, 2.9 and 2.36, (iii) the radius of curvature of R=10 m, (iv) the core temperature of 350 K, (v) the duct walls temperature of 300 K.

The distributions of the volumetric entropy generation due to the heat transfer irreversibility (S_T''') and the fluid frictional irreversibility (S_P''') and total volumetric entropy generation (S_{gen}''') are presented in Figs. 2-9 for better understanding the development of entropy generation in the concentric curved annular square duct by considering different values of the annulus dimension ratio and Dean number. As seen in Figs. 2-5, the effect of fluid frictional irreversibility is highly lower than heat transfer irreversibility (Figs. 6-9) on the volumetric entropy generation. Because the gradient of u and v velocities (Eq. (8)) on the cross-section of curved annular duct are highly smaller than temperature gradient. The maximum values of the Dean number are 207.1 for a/b=5.5, 176 for a/b=3.8, 156.4 for a/b=2.9, and 129.3 for a/b=2.36. Here, the maximum value of Dean number indicates that the flow may become turbulent. When the maximum value of the S_P''' $<10^{-6}$ (De<103.3 for a/b=5.5, De<101.3 for a/b=3.8, De<101.4 for a/b=2.9 and De<78.3 for a/b=2.36) is considered, the variation of volumetric entropy generation due to fluid frictional irreversibility has not been presented in consideration of heat transfer irreversibility because of high temperature gradient. On the other hand, it can be said that the variation of volumetric entropy generation in the concentric curved annular ducts due to fluid frictional irreversibility is directly affected by the secondary flow created by centrifugal effects in the cross-section. The nature of this phenomenon depends upon the Dean number, which is the ratio of the Reynolds number to the square root of the dimensionless radius of curvature. While the fluid flows inside the channel, it is affected by centrifugal force generated by the curvature; thus, the fluid particles move toward the right hand concave wall of the channel. The significant S_P''' appears only in the limited region adjacent to the upper and lower walls of the core and duct because of the high velocity gradient in these regions whereas S_P''' is very minor near the left and right side walls of core and channel and left side gap of the annular duct cross-section. Besides, S_P''' in these regions near the upper and lower walls is found to be larger than that in the region near the left and right side walls of core and channel because of secondary flow resulting from curvature [6]. Also, it is seen that from Figs. 2-5, the highest volumetric entropy generation due to fluid frictional irreversibility occurs near the upper and lower walls of the channel. When Dean number increases, the volumetric entropy generation due to the fluid frictional irreversibility increases (Figs. 2-5) because of the secondary flow occurring in the cross-section [6]. Serious fluid friction occurs in case of the increase of Dean number on the cross-section of concentric curved annular duct and higher frictional irreversibility is induced by the larger De number. As the effect of secondary flow de-

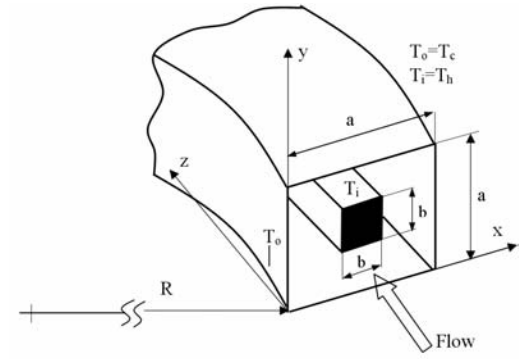

Fig. 1. Problem geometry and coordinate system.

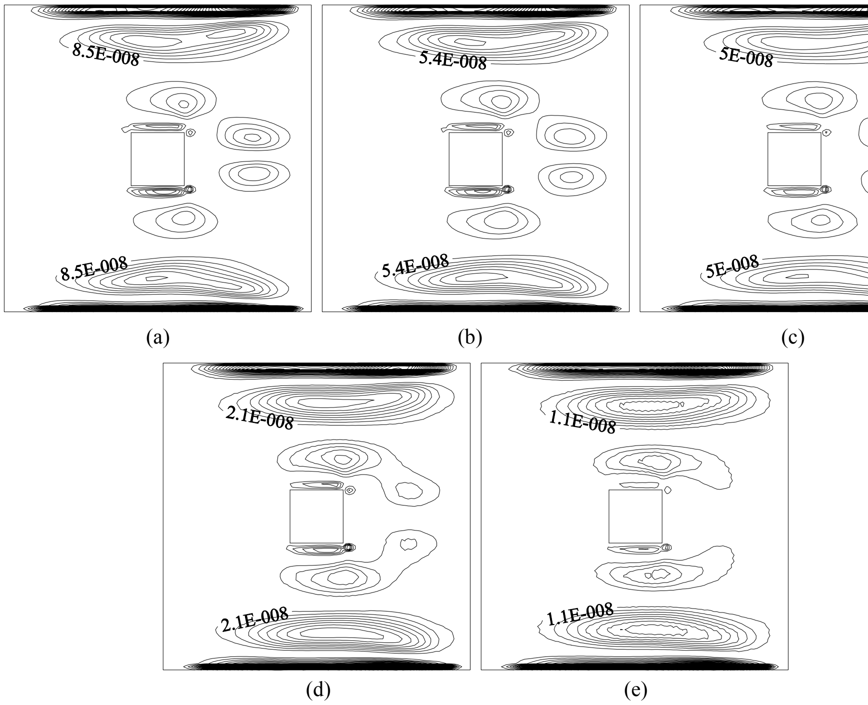

Fig. 2. The contours of volumetric entropy generation due to the fluid frictional irreversibility; a/b=5.5; min:0, (a) De=207.1, max:8.5x10^{-6}, interval:8.5x10^{-8} (b) De=176.0, max:5.4x10^{-6}, interval:5.4x10^{-8} (c) De=156.5, max:3.8x10^{-6}, interval:5x10^{-8} (d) De=129.1, max:2.1x10^{-6}, interval:2.1x10^{-8} (e) De=103.3, max:1.1x10^{-6}, interval:1.1x10^{-8}.

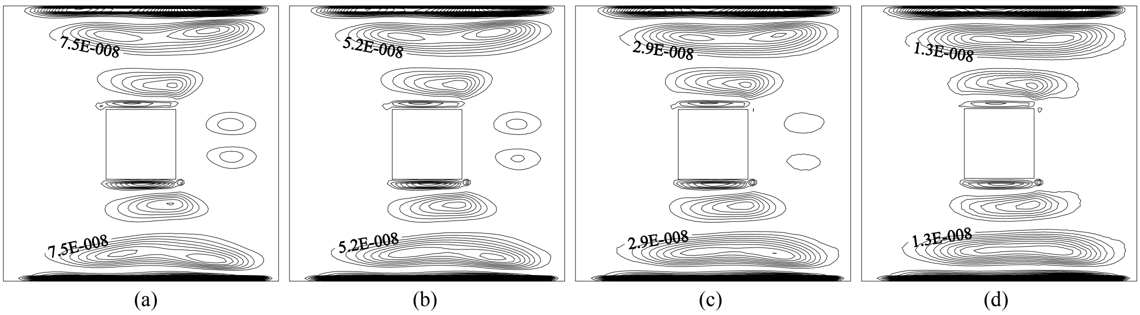

Fig. 3. The contours of volumetric entropy generation due to the fluid frictional irreversibility; a/b=3.8; min:0, (a) De=176.0, max:7.5x10^{-6}, interval:7.5x10^{-8} (b) De=156.6, max:5.2x10^{-6}, interval:5.2x10^{-8} (c) De=129.1, max:2.9x10^{-6}, interval:2.9x10^{-8} (d) De=101.3, max:1.3x10^{-6}, interval: 1.3x10^{-8}.

creases (De<101 for a/b=5.5, 3.8 and 2.9 and De<78.3 for a/b=2.36) the entropy generation highly decreases. When Dean number approaches zero, the curved channel is transformed to straight channel and the gradient of u and v velocities in y and x directions (Eq. (8)) on the cross-section of the annular channel disappear so, the entropy generation resulting from fluid flow irreversibility is zero. When the annulus dimension ratio (a/b) decreases (meaning the small gap between the outer and inner walls) volumetric entropy generation due to fluid frictional irreversibility increases (Figs. 2(d), 3(c), 4(b), and 5(a)). Because when the annulus dimension ratio decreases, the fluid friction so the frictional irreversibility increase.

The distributions of S_T''' and S_{gen}''' on the cross-sectional concentric curved annular square ducts for different annulus dimension ratios (a/b=5.5, 3.8, 2.9 and 2.36) and different values of Dean number are shown in Figs. 6-9. The distributions of the volumetric entropy generation due to the heat transfer irreversibility (S_T''') and total volumetric entropy generation (S_{gen}''') are the same because of very small entropy generation resulting from fluid frictional irreversibility. As described in Eq. (9), entropy can be generated due to the irreversibility of fluid friction and the heat transfer. When Dean number increases, both of the fluid friction and heat transfer coefficient will increase. The volumetric entropy generation due to the heat transfer irreversibility creates the volumetric entropy generation in the concentric curved annular square ducts exposed to the constant surface temperature because of the high temperature gradient. The inner walls (walls of the core) are hot (350 K) and the outer walls (walls of the duct)

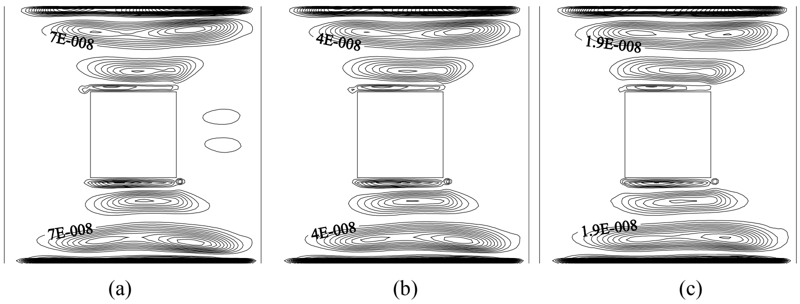

Fig. 4. The contours of volumetric entropy generation due to the fluid frictional irreversibility; a/b=2.9; min:0, (a) De=156.4, max:7.0×10^{-6}, interval:7.0×10^{-8} (b) De=129.2, max:4.0×10^{-6}, interval:4.0×10^{-8} (c) De=101.4, max:1.9×10^{-6}, interval:1.9×10^{-8}.

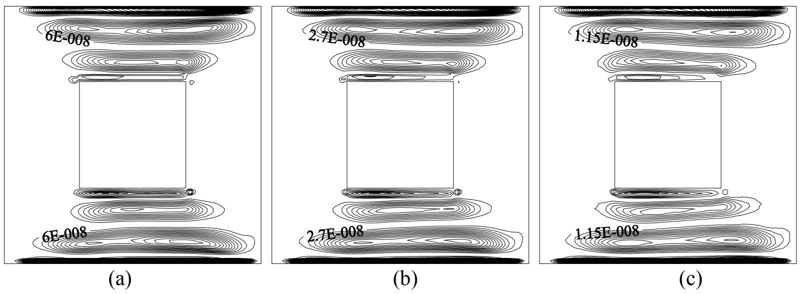

Fig. 5. The contours of volumetric entropy generation due to the fluid frictional irreversibility; a/b=2.36; min:0, (a) De=129.3, max:6.0×10^{-6}, interval:6.0×10^{-8} (b) De=101.3, max:2.7×10^{-6}, interval:2.7×10^{-8} (c) De=78.3, max:1.15×10^{-6}, interval:1.15×10^{-8}.

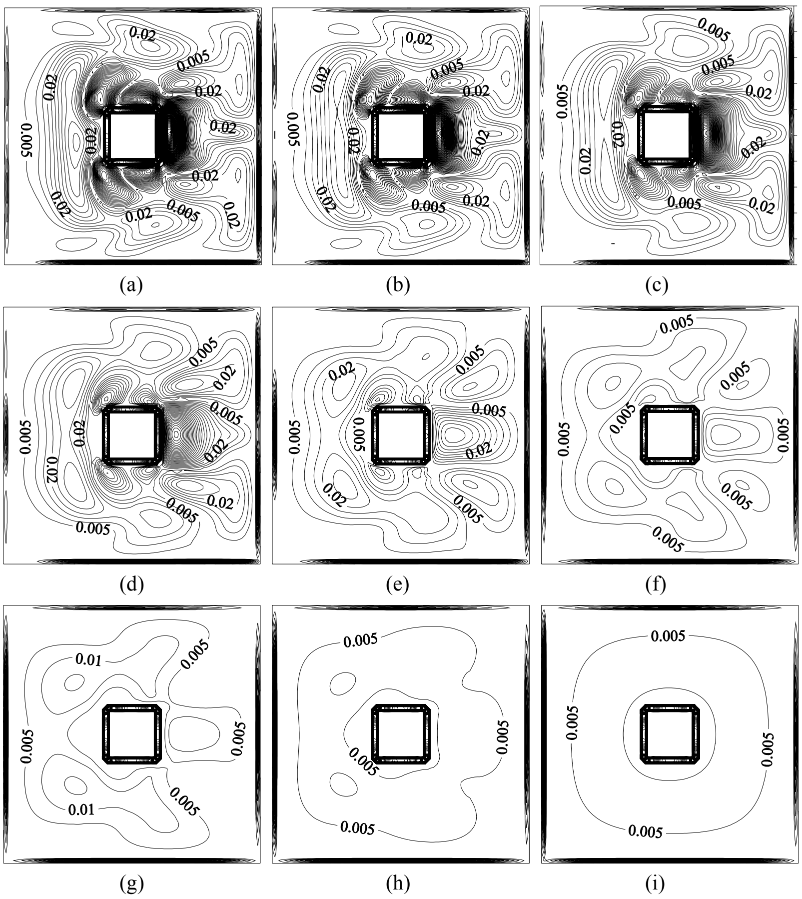

Fig. 6. The contours of volumetric entropy generation due to the heat transfer irreversibility and the total volumetric entropy generation; a/b=5.5; min:0, interval:0.005 (a) De=207.1, max:1.15 (b) De=176.0, max:1.05 (c) De=156.5, max:1.0 (d) De=129.1, max:0.8 (e) De=103.3, max:0.7 (f) De=74.7, max:0.65 (g) De=52.8, max:0.65 (h) De=28.8, max:0.65, (i) De=4.1, max:0.6.

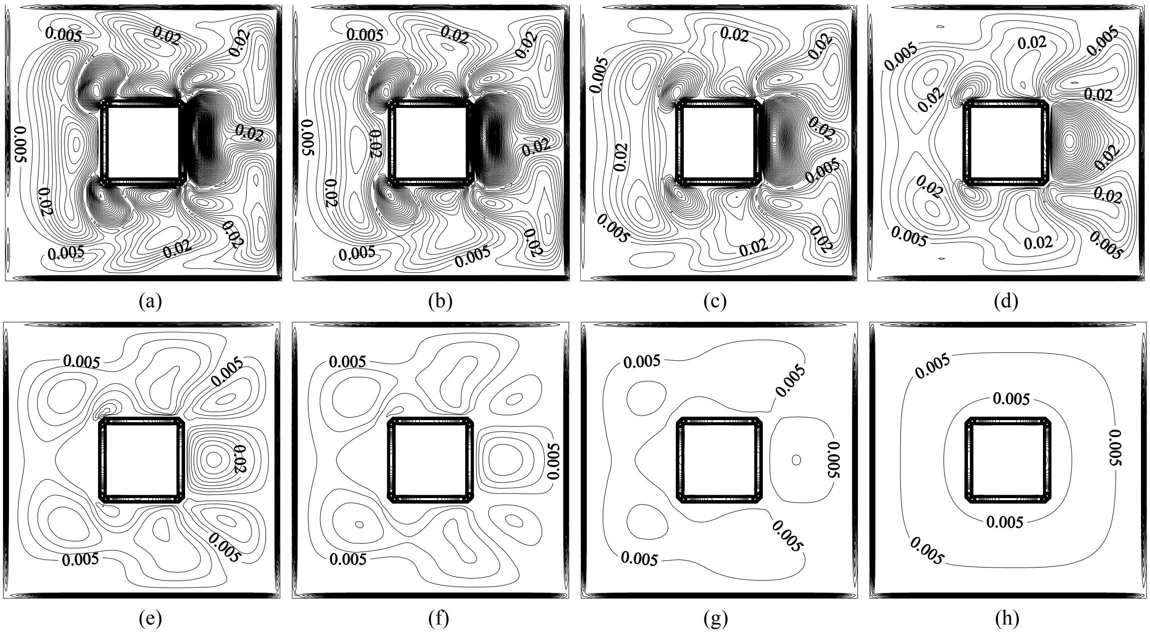

Fig. 7. The contours of volumetric entropy generation due to the heat transfer irreversibility and the total volumetric entropy generation; a/b=3.8; min:0, interval:0.005 (a) De=176.0, max:0.95 (b) De=156.6, max:0.9 (c) De=129.1, max:0.8 (d) De=101.3, max:0.7 (e) De=74.8, max:0.65, (f) De=57.6, max:0.65 (g) De=28.0, max:0.6 (h) De=2.8, max:0.6.

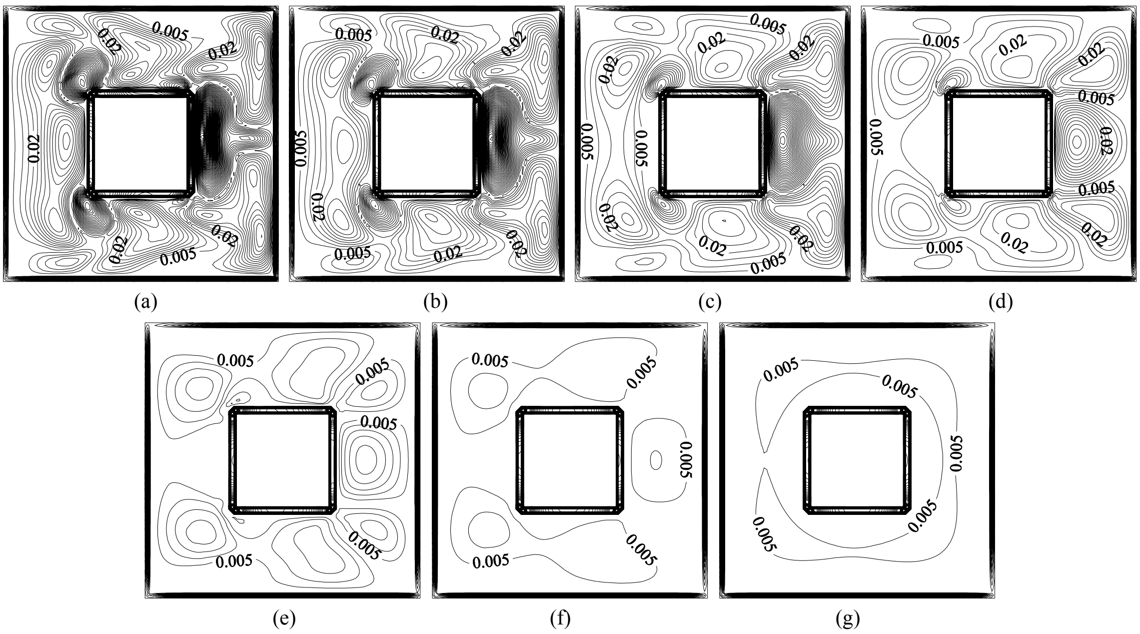

Fig. 8. The contours of volumetric entropy generation due to the heat transfer irreversibility and the total volumetric entropy generation; a/b=2.9; min:0, interval:0.005 (a) De=156.4, max:0.8 (b) De=129.2, max:0.75 (c) De=101.4, max:0.65 (d) De=78.3, max:0.65 (e) De=54.1, max:0.65 (f) De=27.4, max:0.6 (g) De=3.6, max:0.6.

are cold (300 K). The distribution of significant S_T''' only occurs in the region close to core and duct walls where the temperature are most significant, whereas S_T''' is small in other regions of the duct. The most significant S_T''' concentrates near the core walls exposed to the 350 K. The influences of Dean number on volumetric entropy generation due to heat transfer irreversibility and total volumetric entropy generation are investigated through the comparison of Figs. 6-9 for a/b=5.5, 3.8, 2.9 and 2.36, respectively. When Dean number has the lowest value (almost straight channel) the entropy generation is maximum near the core and duct walls and very small other regions of cross-section of the duct for all annulus dimension ratio. As Dean number increases the distribution of volumetric entropy generation resulting from heat transfer irreversibility and total volumetric entropy generation is formed by the temperature field depending on the curvature

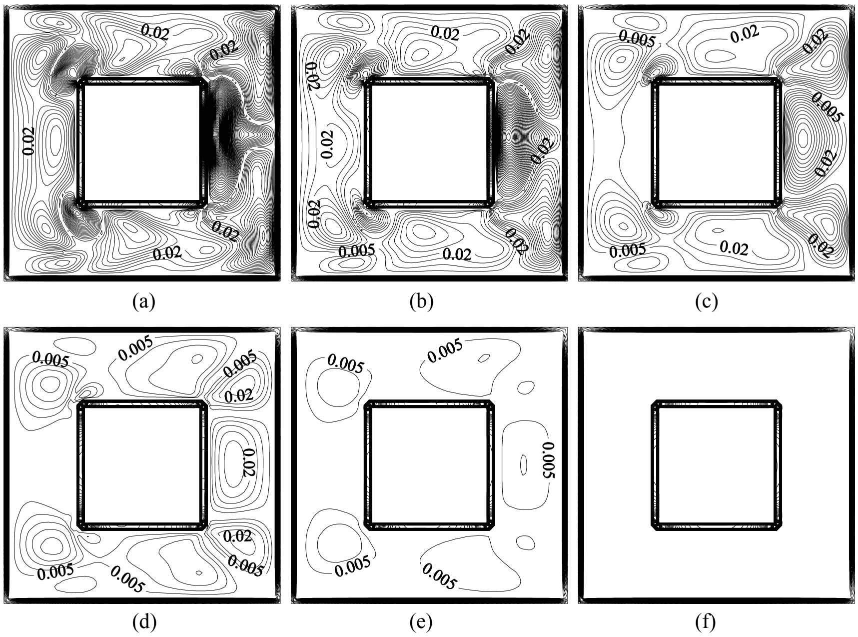

Fig. 9. The contours of volumetric entropy generation due to the heat transfer irreversibility and the total volumetric entropy generation; a/b=2.36; min:0, interval:0.005 (a) De=129.3, max:0.75 (b) De=101.3, max:0.65 (c) De=78.3, max:0.65 (d) De=54.4, max:0.65 (e) De=29.6, max:0.65 (f) De=4.5, max:0.6.

[6]. It is seen that the maximum entropy generation increases when the Dean number increases. Also, it is observed that the maximum entropy generation occurs near the right wall of the core because of maximum temperature gradient resulting from centrifugal force generated by the curvature as Dean number increases. The contours of entropy generation become intensive when Dean number increase. The entropy generation near the left wall of the channel decreases as Dean number increases because of low temperature gradient in this region. The contours of entropy generation also become dense when the annulus dimension ratio decreases (meaning the small gap between the outer and inner walls).

The variation of average volumetric entropy generation resulting from fluid frictional irreversibility computed from Eq. (12) is given in Fig. 10 depending on Dean number. It is seen that the volumetric entropy generation due to fluid frictional irreversibility is very low in concentric curved annular duct under constant wall temperature boundary condition for laminar flow because of very low gradient of u and v velocities (Eq. (8)) occurring on the cross-section. At the lowest Dean number, entropy generation is almost zero for each annulus dimension ratio in case of the fluid frictional irreversibility. When Dean number increases the entropy generation resulting from fluid frictional irreversibility increases because of curvature creating centrifugal force. Centrifugal force causes the secondary flows, which are increasing the friction, in the cross-section of the concentric curved annular duct [6]. Also, it is observed that when the annulus dimension ratio decreases (meaning the small gap between the outer and inner walls) the volumetric entropy generation due to fluid frictional irreversibility increases because as the annulus dimension ratio decreases the friction increases in the cross-section of the channel.

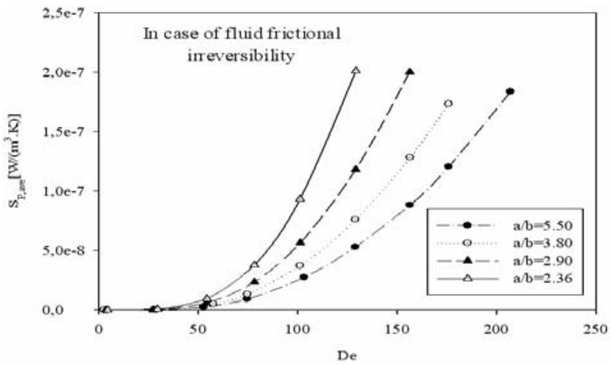

Fig. 10. The variation of average volumetric entropy generation due to the fluid frictional irreversibility with Dean number.

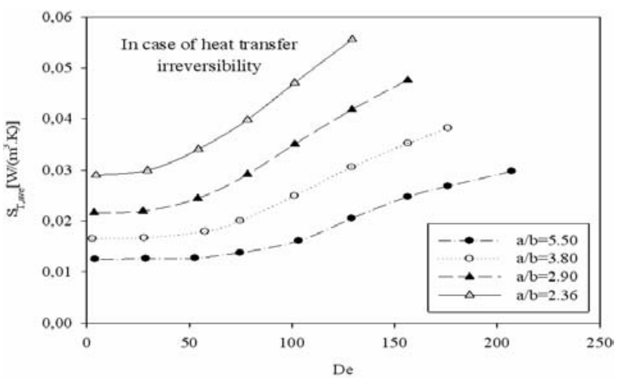

Fig. 11. The variation of average volumetric entropy generation due to the heat transfer irreversibility with Dean number.

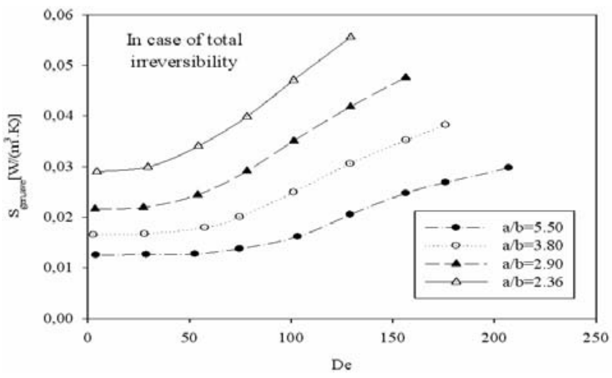

Fig. 12. The variation of average total volumetric entropy generation with Dean number.

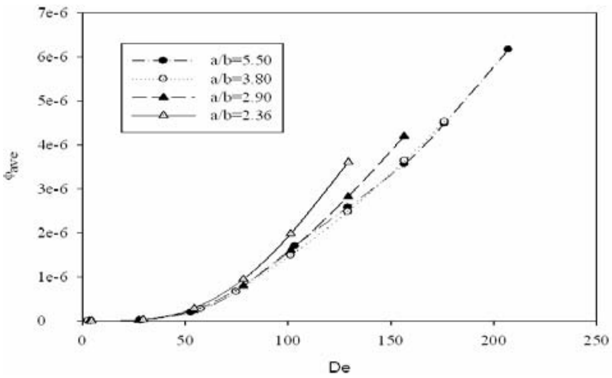

Fig. 13. The variation of average irreversibility distribution ratio with Dean number.

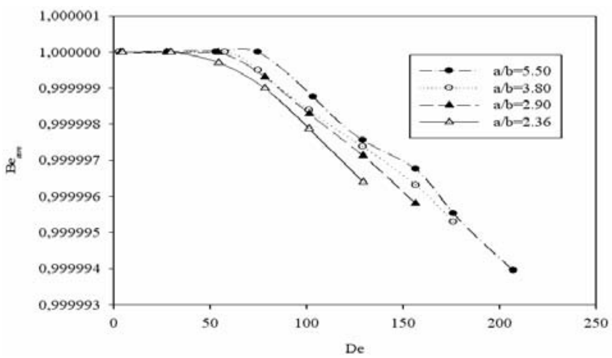

Fig. 14. The variation of average Bejan number with Dean number.

The variations of average volumetric entropy generation due to the heat transfer irreversibility computed from Eq. (13) and total volumetric entropy generation obtained by Eq. (14) are shown in Figs. 11 and 12, respectively, depending on Dean number. Total volumetric entropy generation is equal to sum of volumetric entropy generation resulting from fluid frictional irreversibility and heat transfer irreversibility as given in Eq. (9). As entropy generation due to fluid frictional irreversibility is too small, the entropy generation due to heat transfer is almost equal to total entropy generation as seen in Figs. 11 and 12. Because of high temperature gradient, the entropy generation resulting from heat transfer irreversibility creates all entropy generation in the cross-section of curved annular square channel under constant wall temperature boundary condition for laminar flow. As seen in Figs. 11 and 12, both entropy generations increase when Dean number increase because of secondary flows. Also, it is seen that when the annulus dimension ratio decreases (meaning the small gap between the outer and inner walls), total volumetric entropy generation and entropy generation due to heat transfer irreversibility highly increases.

The variation of average irreversibility distribution ratio is illustrated as in Fig. 13. It is seen that the irreversibility distribution ratio is very low so, the effect of volumetric entropy generation resulting from fluid frictional irreversibility can be neglect as compared with volumetric entropy generation due to heat transfer irreversibility in concentric curved annular ducts under constant wall temperature boundary condition for laminar flow. Also, it is observed that irreversibility distribution ratio increases when Dean number increases. Moreover, as Dean number increases, the increase of irreversibility distribution ratio is high for a/b=2.36 and 2.9, particularly. It can be said that the increase of entropy generation due to fluid frictional irreversibility is higher than that of entropy generation due to heat transfer irreversibility when Dean number increases. Moreover, the rise of friction with curvature creating centrifugal force is higher than that of heat transfer in curved annular duct for fully developed laminar flow.

The variation of the average Bejan number is presented in Fig. 14. It is seen that the average Bejan number is almost equal to 1 for all annulus dimension ratios and Dean numbers. It can be said that the volumetric entropy generation resulting from heat transfer irreversibility is completely dominant on the cross-section of concentric curved annular ducts under constant wall temperature boundary condition for laminar flow. When Dean number increases and annulus dimension ratio decreases (meaning the small gap between the outer and inner walls) Bejan number decreases. Because influence of heat transfer irreversibility decreases as curvature creating secondary flows resulting from centrifugal force increases and annulus dimension ratio decreases, which generally increases friction than heat transfer.

5. Conclusions

The entropy generation for hydrodynamically and thermally fully developed, steady, incompressible laminar flow with constant physical properties in the concentric curved annular duct with square cross-section was numerically investigated under constant wall temperature boundary condition.

The effect of fluid frictional irreversibility is highly lower than heat transfer irreversibility on volumetric entropy generation for laminar flow in concentric curved annular square duct under constant wall temperature boundary condition. The variation of volumetric entropy generation due to fluid frictional irreversibility in the concentric curved annular ducts is directly affected by the secondary flow created by centrifugal effects in the cross-section. The significant S_P''' appears only in the limited region adjacent to the upper and lower walls of the core and duct because of the high velocity gradient in these regions. Serious fluid friction occurs by the increase of Dean number on the cross-section of concentric curved annular duct and higher frictional irreversibility is induced by the larger De number (De≥101 for a/b=5.5, 3.8 and 2.9 and De≥78.3 for a/b=2.36). When the annulus dimension ratio (a/b) decreases the volumetric entropy generation due to fluid frictional irreversibility increases.

The volumetric entropy generation due to the heat transfer irreversibility creates the volumetric entropy generation in the concentric curved annular square ducts exposed to constant surface temperature because of high temperature gradient for laminar flow. The distribution of significant $S_T^{'''}$ only occurs in the region close to core and duct walls where the temperature is most significant. The most significant $S_T^{'''}$ concentrates near the core walls exposed to the 350 K. As Dean number increases the distribution of volumetric entropy generation resulting from heat transfer irreversibility and total volumetric entropy generation is formed by the temperature field depending on the curvature under constant wall temperature boundary condition. The maximum entropy generation occurs near the right wall of the core because of maximum temperature gradient resulting from centrifugal force generated by the curvature as Dean number increases. The contours of entropy generation also become dense when the annulus dimension ratio decreases.

The effect of volumetric entropy generation resulting from fluid frictional irreversibility can be neglect as compared with volumetric entropy generation due to heat transfer irreversibility in concentric curved annular square ducts under constant wall temperature boundary condition for laminar flow.

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